SEMESTER 5

Semester 5	(UE)	nb of hours	CM	TD	TP
Scientific knowledge			**65**	**65**	
IMathematical methods for engineers	**4**	60	30	30	
Numerical analysis I	**4**	70	35	25	10
Methods and techniques			**106**	**73**	**51**
Algorithmie+Unix+Outil du Calcul Scientifique (Fortran90, verification/validation de code)+git	**4**	70	35	14	21
optimisation continue	**4**	60	25	25	10
Probabilité+ Chaine de Markov+ simulaton aleatoire	**6**	100	46	34	20

MATHEMATICAL METHODS FOR ENGINEERS
(30HCM, 30HTD, 4 ECTS)

Educational purpose:

- Functions of a complex variable, Fourier transforms, separation of variables Laplace transforms (10 HCM, 10HTD)
- Vectorial calculus (Green and Ostrogradsky's theorems) (5HCM, 5HTD)
- Theory of integration (Lebesgue measures, LP spaces, distributions and basic properties , Sobolev spaces) (15HCM -15HTD)

NUMERICAL ANALYSIS I
(35HCM, 25HTD, 10HTP, 4 ECTS)

Educational purpose : To acquire the basic numerical methods used to solve linear and non-linear problems (with finite difference dicretizations)

Content :

1. Finite difference discretization : Basic principles (Taylor's formula), 2D and 1D examples (explanation of the numbering 2D->1D, use of maple), Richardson's extrapolation.
2. Direct methods: LDU factorization (Existence and uniqueness conditions of the decomposition, matrix conditioning, impact of conditioning on the error, pivoting technique, computation of the determinant),profile conservation (application to periodic boundary conditions), Schur complement and its properties.
3. Numerical interpolation and integration: quadrature formulas (Newton-Cotes, Romberg's methods), Gibbs phenomenon, orthogonal polynomials , Hermite's interpolation, (Divided differences formula and associated error estimation) Interpolation using rational functions (Padé approximant)
4. Newton's methods/fixed point iteration: Basic priciples (fixed-point method , Newton's method),Affine invariance theorem, quasi -Newton methods
5. Basic iterative methods: splitting methods: (Jacobi, Gauss-Seidel, ,SOR), General convergence theorems (based on spectral radius, optima SOR), Krylov's methods: (principles, GC, GMRES), Convergence theorems based on eigenvalue distribution
6. Solution methods for ODE's: One-step methods/Theory (consistency, stability, convergence impact of round-off errors, Grownwall's lemma, Euler methods, implicit, explicit, mid-point, adaptive step size methods, stiff problems), Introduction to multistep schemes (BDF)

References:

J.Stoer & R. Bulirsh: Introduction to numerical analysis, Springer text in applied Math. 15

G.H. Golub & C.F. Van Loan: Matrix computation, J. Hopkins University press

E. Hairer & G. Wanner : Solving Differential Equations I, Springer series in Comput. Math 14

Y. Saad : Iterative methods for sparse linear systems, SIAM

COMPUTATIONAL TOOLS
(35HCM, 14HTD, 21HTP, 4 ECTS)

Educational purpose :

Mastery of basic of algorithmics,Unix command of version control systems, imperative programming, code verification and validation techniques

Prerequisite : Numerical analysis I

Content:

1. Algorithmic structures:identifiers arithmetic and boolean expressions, statements: syntax and semantics,standard algorithmic constructions. (6HCM+6HTD)
2. Unix: on line help, file system environment variable,files handling commands makefile and shellscripts creation, . (3HCM/3HTP)
3. Version control systems:Introduction to GIT: create a repository ,commit sources,retrieve sources from a repository,create development branches, go back to previous version. (2HCM/2HTP)
4. Finite arithmetics of computers and their consequences : computation conditioning, backward or a posteriori analysis. (2HCM/2HTD)
5. Imperative programming: Fortran 90, Tables,complex data structures, procedures and functions, recursion, dynamic allocation, module. Implementation of numerical analysis methods I(CSR stocking, iterative methods and Krylov's method). Introduction to BLAS and LAPACK scientific libraries . (16HCM/16HTP)
6. Code verification: software quality certification (static and dynamic analysis, unit testing, consistency and convergence, formal and observed accuracy order , manufactured solutions methods .(4HCM/4HTD)
7. Code validation: good practices guide, entrained cavity example(2HCM/2HTD)

Support software : Fortran90, BLAS, LAPACK, Maple

References:

R. Séroul: Programming for Mathematicians, springer 1995

M. Metcalf, J.Reid, M.Cohen: Fortran 95/2003 explained, Oxford University press, 2007

P. Roache: Verification and Validation in Computational Science and Engineering, Hermosa publishers, 1998

<u>CONTINUOUS OPTIMIZATION</u>

(25h CM, 25h TD, 10h TP ; 4 ECTS)

Educational purpose :

To recognize an optimization problem : to deal with the existence of solutions to the problem and provide (an) approximate value(s) of the solution(s) using appropriate numerical methods

Content :

Optimization without constraints and with equality or inequality constraints . Extremum conditions.Solution of $F(X)=0$ using Newton and quasi-Newton methods; application to optimization. Lagrange multipliers, saddle point, duality.
Software support : Matlab.

PROBABILITY AND SIMULATION OF RANDOM EVENTS
(50h CM,24h TD,16h TP, 6 ECTS)
DESCRIPTIVE STATISTICS WITH R - 4h CM , 6h TP
Educational purpose
By the end of the course, students are expected to be able to analyze a random variable using standard parameters as well as the interaction between 2 such variables. They will also master the R software.
Content
Satndard parameters : mode, mean, median, quantiles, range, standard deviation, kurtosis, skewness
Relevant graphs : boxplots, histograms, pie charts ;
Relationship between 2 variables : covariance, correlation, cindependence ;
Software R : handling of data, obtain basic statistical parameters and graphs .
Support software : R
References
Cornillon, P.A.et al., *Statistiques avec R*, Presse Universitaire de France, 2010.
Grais, B., *Méthodes statistiques*, Dunod, 2003.

PROBABILITY THEORY - 30h CM, 30h TD
Educational Purpose
By the end of the course, students are expected to be able to handle and find out probability distributions and to identify the common ones . Besides they are also expected to be conversant enough with stochastic convergence and conditioning and related theorems to be able to apply them in the various subsequent probability and statistics courses.This course is essential to the understanding of courses dealing with random modeling.
Prerequisite: Mathematical methods for engineers.
Content
Conditional probability, independence ;
Random variable : common distributions , variable transformation, quantiles, moments ;

Random vectors, gaussian vector ;
Laplace transform, characteristic function ; conditional expectation ;
Various types of convergence, law of large numbers, central limit theorem ..

References

Foata, D. & Fuchs, A. *Calcul des probabilités, cours, exercices et problèmes corrigés*, Dunod, 1998.
Jacod, J. & Protter, P., *L'essentiel en probabilités ou bien les bases de la théorie des probabilités.*, Cassini, 2003.
Billingsley, P., *Probability and measure*,Wiley, 1995.
Grimmet, G.R.& Stirzaker, D.R., *Probability and Random Processes*, Oxford Science Publications, 1992.

RANDOM SIMULATION PART I - 10h TP

Semester 6		nb of hours	CM	TD	TP
Scientific knowledge			**60**	**60**	
Statistique inférentielle	**4**	60	30	30	
Modèles Mathématiques pour système multi-physiques (Electro/thermo/automatique/mecaflux/Elasticité)	**4**	60	30	30	
Methods and techniques			**104**	**20**	**116**
Genie logiciel (Programmation Orienté Objet+UML Lite)	**4**	60	28	4	28
Recherche Opérationelle et CAO	**3**	60	25	10	25
Analyse de données (SAS) et base de donnée	**4**	72	34		38
Simulation aléatoire en statistique (logiciel R)	**3**	48	22		26

Educational purpose

By the end of the course, students should be able to simulate random variable distributions and will have put their knowledge of probability theory into practice.

Prerequisite: Course in probability.

Content

Random variables simulation :inverse distribution function, acceptance/rejection...
Monte Carlo methods ;
Hands-on exercises relating to the course in probability

Support software : R

References

Bouleau,N., *Probabilités de l'Ingénieur, variables aléatoires et simulation*, Hermann, 2002. Devroye, L., *Non-Uniform Random Variate Generation*, Springer, 1986.
Robert, C. et Casella, G., *Monte Carlo StatisticalMethods*, Springer-Verlag, 2004.

MARKOV CHAINS - 12h CM, 4h TD, 4h TP

Educational purpose

This course is meant to teach the fundamentals of markovian phenomena modeling.By the end of the course, students will have been given the opportunity to put the above concepts into practice (reliability,waiting lines, genetics, economics..)

Prerequisite Probability theory, Random simulation Part I.

Content

Definition, transition matrix and standard examples.
Chapman-Kolmogorov equation and conditioning.
Classification of states, periodicity, waiting time, recurrence and transience.
Steady-state distribution and limit theorems.

Support software : R

References

Pardoux, E., *Processus de Markov et applications : Algorithmes, réseaux, génome et finance.* Dunod, 2007.
Foata, D. & Fuchs, A., *Processus stochastiques - Processus de Poisson, chaînes deMarkov et martingales, Cours, exercices et problèmes corrigés*, Dunod, 2002.
Bercu, B & Chafai, D., *Modélisation stochastique et simulation*, Dunod, 2007.

Semester 6

INFERENTIAL STATISTICS
(30h CM, 30h TD, 4 Ects)

Educational purpose: By the end of the course, students should master statistical modeling besides being be able to build estimators and study their properties.They should have become proficient in the construction of confidence intervals and in the use of standard statistical tests, whether parametric or not, both from a theoretical and practical perspective.

Prerequisite: Probability theory

Content

Statistical models ;
Parametric estimation : insertion method, qualities of an estimator ;
¦ likelihood estimation : maximum likelihood, Fisher information, properties of the maximum likelihood estimator ¦ confidence interval ;
¦ statistical tests :associated risks, optimality, asymptotic test,likelihood-ratio test, independence and suitability test ;
¦ Gaussian model : Cochran's theorem, multiple linear model, Fisher's test, means comparison test ;
¦ Applications : statistical methods in Markov chains

References

¦ Rivoirard, V. & Stoltz, G., *Statistique en action*, Vuibert, 2009.
¦ Fourdrinier, D., *Statistique inférentielle-Cours et exercices corrigés*, Dunod, 2002.

MULTIPHYSICAL MODELING
(ELECTRO/THERMO/AUTOMATICS/FLUIDMECHNICS/ELASTICITY
(30 h CM, 30h TD, 4 ECTS)

Educational purpose:

1. Understanding the mathematical models of multiphysics systems, the classification of variables and the relationships between them

2. Hamiltonian formulation of dynamic models of open physical systems and of their properties.
3. Covariant formulation of conservation laws and spatial discretization methods, while keeping their symplectic or Dirac structure .
4. Controllability and stabilization using Lyapounov's methods
5. Simulation and control using softwares (Matlab®, Scilab® ...).

Prerequisite: Basic knowledge of differential and vector calculus , Fourier and Laplace transforms.

Content

The course consists of 4 parts.

- The first part deals with macroscopic multi-physics systems which are governed by systems of conservation laws with source terms (balance laws) .A structural classification of such systems is given according to the nature of their variables (storage variables , fluxes, driving forces, thermodynamic state variables) and of the relationships between them.This structure will be highlighted by different applications pertaining to heat, mass and momentum transfer, to elastodynamics, fluid mechanics and electromagnetism.
-
- The second part addresses the Lagrangian and Hamiltonian formulations of open multi-physics systems and of their coupling.Finite and infinite dimensional port-Hamiltonian systems with dissipation will be introduced as well as their interconnection and the associated composition of Dirac structures.Properties of such systems are discussed: dynamic invariants, dissipativity,...
-
- The third part discusses the properties of the systems of PDE's associated with the above models and introduces both the differential forms and the covariant formulation of the conservation laws. It also deals with ways to adapt finite element discretization schemes and pseudo-spectral methods preserving the hamiltonian structure of dynamical systems.
- The fourth part which is devoted to Continuous automatic control, presents controlled systems and their structural properties, along with feedback command and state variable feedback control.The state approach issue is essentially dealt with, relying on a system of 1rst order ODE's, on the controllability and observability properties and on the state variable control synthesis.Where non-linear systems are concerned, control Lyapunov functions are introduced since they are particularly suited to dissipative port-Hamiltonian systems.

- **References:**

<u>RANDOM SIMULATION PART II</u>
(22H CM, 26H TP, 3 ECTS)
Educational purpose:
This course first aims at putting the main results of Inferential Statistics into practice but , in a second step, also touches upon advanced statistical methods (non-parametric statistics for example), which prove to be very useful, such as Bootstrap's methods.
Prerequisite: Probability theory, Inferential statistics, Random simulation Part I
Content
- Simulations related to the course in Inferential Statistics : point and interval estimation, parametric and non-parametric tests (CM 2h, TP 10h) ;
- parametric and non-parametric Bootstrap (ex :Estimation algorithms for Bias/Variance/Mean squared error/Confidence intervals)(CM 10h, TP 10h);
- Non parametric statistics (density estimation, regression), rank test (CM 10h, TP 6h).

Software support : R
References
Silvermann, B.W., *Density Estimation for Statistics and Data Analysis*, Chapman & Hall, 1998.
Tsybakov, A. B., *Introduction à l'estimation non-paramétrique*, Springer, 2004.
Efron B.& Tibshirani, R. J., *An introduction to the bootstrap*, 1994.
Van der Vaart, A.W. *Asymptotic Statistics* Cambridge university press, 1998.

<u>DATA ANALYSIS AND DATABASE</u>
(34h CM, 38h TP, 4 ECTS)

DATA ANALYSIS AND CLASSIFICATION - 24h cours, 28h TP en SAS

Educational Purpose
By the end of the course, students should be able to put the standard data analysis methods into practice and should have gained some proficiency in the use of the SAS software.
Prerequiqite : Descriptive statistics using R.
Content

- Introduction to SAS (Data stage, procedure stage, graphical procedures and first elementary stastics procedures) (CM 10h, TP 10h);
- Data analysis (CM 16h, TP 18h)
  - Principal component analysis(PCA) ;
  - Factorial correspondence analysis (FCA)
  - Multiple correspondence analysis (CAM);Classification method ;Discriminant analysis (decision and exploratory analysis)
  - **Support software : SAS**

References
Droesbeke, J.J & Fichet, B. & Tassi, P., *Modèles pour l'analyse des données multidimensionnelles*, Economica, 1992.

Semester 7		nb of hours	CM	TD	TP
STAGE S7					
	30	STAGE 1 Septembre 15 Janvier			

Escofier, B. & Pages, J., *Analyses factorielles simples et multiples*, Dunod, 1998.
Husson, F. & Le, S. & Pages J., *Analyse de données avec R.* Presses Universitaire de Rennes, 2009.

DATABASES - (10h CM - 10h TD)
Educational purpose :
To acquire the basics of database manipulation in order to apply them to data-mining and risk analysis.

Prerequisite: Basic knowledge of algorithmics and statistics.
Content :

Semester 8			CM	TD	TP	HP
Scientific knowledge			**26**	**14**	**20**	
Modelisation en statistique	**4**	60	26	14	20	
Methods and techniques			**125**	**85**	**60**	**30**
Méthode de discretisations	**5**	85	38	32	5	10
Analyse Numerique II	**4**	70	25	20	15	10
Problème instationnaires	**5**	75	30	15	15	
Serie temporelle et modèle de durée	**4**	70	30	20	10	10

Storage of huge amounts of complex data/introduction to its principles– Data search and sorting within a large data set.Database organization. Object databases. Basic knowledge of SQL.
Support software : Access, Spad.

DISCRETE OPTIMIZATION & CAD
(25h CM, 10h TD, 25h TP ; 3 ECTS)

DISCRETE OPTIMIZATION (10h CM, 10h TD, 10h TP)
Educational purpose :
To enable students to implement algorithmic solutions to graph type problems.
Prerequisite : Basic algorithmics
Education purpose : Linear programming, simplex. Integer programming. Assessment and separation procedures,heuristic procedures. Annealing and genetic algorithms . Graphs ; pathfinding, flows, cover tree, maximum coupling. Introduction to scheduling methods
Support software : Matlab

CAD (15h CM, 15h TP)
Educational purpose:
To acquire the basic concepts involved in geometry manipulation in CAD. To learn about the different existing design models.To implement a CAD code used in industry. An upstream intervention stresses the challenges of CAD and its integration within companies.
Prerequisite: Basic knowledge of curves and surfaces. Basic numerical methods and Matlab numerical applications.
Content:

Upstream : Introduction and role of CAD in the iterative design process; Presentation of current CAD systems and of the main suppliers of CAD and PDM.Different design models depending on market shares; Outline of current issues.
Geometric modeling of curves and surfaces : Bézier model : definition of curves ; De Casteljau's algorithm and other algorithms (subdivision, degree elevation…) ; Bézier patches ; Sticking patches together and geometric continuity. Shortcomings of the model. B-Spline Model, De BOOR's evaluation algorithm, geometric properties and limitations.
NURBS Model : curves and surfaces. Aspects of « algorithmic geometry» : shape triangulation, search for convex envelopes for the calculation of intersections. Other geometric design models (CSG, BREP). Exchange formats.
CAD practice.
Support software : CATIA V6 ou ProEngineer.

SOFTWARE ENGINEERING

(28h CM, 4h TD, 28h TP ; 4 ECTS)
Educational purpose: To have a methodological tool available to implement the object approach (UML2.1.1). To be able to implement the concepts of object-oriented programming using the C++ language : to be able to build a simple application and to contribute to a large and complex one.Introduction of the object concept in scientific computation.
Prerequisite: Tools of scientific computing , Numerical analysisI.
Content:
°Management of a complex application code : object solution using UML 2.1.1 (4HCM,2HTD,2HTP) UML methodology concepts (OMG norm (Object Management Group)). Static views of a system:object, class, component, deployment diagrams. Dynamic views of a system: collaboration, sequence,state-transition, activity diagrams

- Object-oriented programming for scientific computing: (16HCM, 16HTP). Basics of POO: object, class, polymorphism, dynamic link, C++ programming : class, operators, friendship concept, generic programming : (template).Simple and multiple Inheritance . STL and scientific libraries implementation (4HCM/6HTP).Structure of advanced data for scientific computing : valarray, map, vector. Introduction to the Template Numerical Toolkit library (lapack++, IML++, SparseLib++, mv++.). Inclusion of new digital features into a complex object-oriented code designed for scientific computing (hydrology, process engineering,…).

°Development of human-machine interfaces (2HCM,2HTD, 2HTP). Use of the Qt library and graphism .

- Code optimization, debugging tolls (2HCM, 2hTP)

Support softwares : Environnement Eclipse Environment, OMONDO/BOUML, STL library, Boost library, GSL 1.9, Template Numerical Toolkit, Java Sun et BEA Logic compilers.

Semester 7

Semester 8

STATISTICAL MODELING

(28 HCM, 14H TD, 18H TP, 4 ECTS)

BAYESIAN MODELS - 8h CM, 8h de TD/TP

Educational purpose

Students are expected to be able to implement a bayesian methodology for standard modelings : from the choice of a prior probability distribution to the construction of a confidence interval.Practical applications will be run using the R software.

Prerequisite: Markov chains, Inferential statistics, Random simulation Parts I et II.

Content

Markov chain Monte Carlo methods (MCMC);
Prior and posterior probability distributions (explicit or through MCMC) ;
Bayesian inference : bayesian risk, estimation,confidence tests and intervals.

Support software : R

References

Robert, C. P., *L'analyse bayésienne*, Sringer, 1992.
Robert, Ch., *Méthodes deMonte Carlo par chaînes deMarkov*, Economica, 1996.Gilks,W. R. & S. Richardson, & D. J. Spiegelhalter, *Markov chainMonte Carlo in practice*, Chapman and Hall, 1996.

REGRESSION MODELS - 20h CM, 24h TD/TP

Educational purpose

By the end of the course, students should be able to perform a linear regression analysis,to critically analyze a software output, to treat atypical points , to choose a suitable model and to master the variance and covariance analysis models. They will be able to implement the various models using SAS and R.

Prerequisite Random modeling of semesters S5 et S6.

Content

Simple and multiple linear regression : model,least square estimators,, gaussian model, confidence interval, tests, prediction,residual analysis, model selection ;
Study of special cases : ANOVA, ANCOVA ;
Support softwares : R et SAS
References
Azaïs J-M. & Bardet J-M., *Le modèle linéaire par l'exemple Régression,*

Semester 9		nb of hours	CM	TD	TP	HP
Scientific knowledge			**45**		**45**	
Modelisation Mathématique-Galerkin discontinu	**5**	90	45		45	
Methods and techniques			**108**	**51**	**51**	**20**
Calcul Haute performance	**4**	70	35		35	
statististique des processus	**3**	50	26	20	4	
Modélisation statistique avancée	**4**	60	32	16	12	
Projet	**6**	80	15	15		50
option Big Data			15	15		
option contrôle			15	15		

analyse de la variance et plans d'expérience illustrés avec R, SAS et Splus, Dunod, 2006.
Matzner-Løber, E. & Cornillon, P-A., *Régression, Théorie et applications*, Springer, 2007.Weisberg, S., *Applied Linear Regression*,Wiley, 2005.
Droesbeke, J.-J., Fine, J., Saporta, G., *Plans d'expériences - Applications à l'entreprise*, Technip, 1997.

DISCRETIZATION METHODS: (38h CM, 32h TD, 5h TP ; 10h HP, 5 ECTS)

Educational purpose:
To learn how to numerically approximate PDE problems (elliptic, linear parabolic and hyperbolic) using the finite element method (FEM)
Application to structural and fluid mechanics: linear elasticity, Navier-Stokes,..
Prerequisite: Numerical analysis I,Mathematical methods for engineers

Content:
Examples of elliptic,parabolic and hyperbolic problems. Variational formulation of elliptic boundary value problems. Description of Sobolev spaces. Lax-Milgram theorem.

Finite element approximation using the examples of diffusion, elasticity, transmission, diffusion-convection problems… Finite element spaces: examples of 1D, triangular, rectangular elements, and extension to 3D.

Stability and convergence analysis for elliptic problems.
(Smoothness of the exact solution and its finite element approximation).

Application to a few elliptic problems in continuum mechanics (Elasticity, Stokes).
(Smoothness of the exact solution and its finite element approximation)

Fundamentals of FEM implementation.Practice of a FEM code actually used in R&D.

Mixed formulation of elliptic problems and Raviart-Thomas type finite element approximation.

Mixed formulation of incompressible fluid flow problems. Stokes problem.Stability and convergence analysis.. Inf-Sup condition (LBB).
Usual finite element spaces and associated convergence results.
Pressure treatment.

Solution of saddle-point problems: uzawa et penalty methods.
Mixed formulations in solid mechanics for plate problems (Hellinger-Reissner-Mindlin, Hu-Washisu approaches,...).
Consideration of the role of non-linearity (convection), transport processes, reaction : Stabilization using Upwind, SUPG, GALS, mass-lumping methods…

Support software :

COMSOL Multiphysics / CAST3M (CEA)

References:

P.G. Ciarlet: The finite element method for elliptic problems. North-Holland 1978.
A. Quarteroni et A. Valli : Numerical Approximation of partial differential equations, Springer-Verlag, Berlin, 1994.
O. Pironneau: Méthode des éléments finis pour les fluides. Collection Mathématiques appliquées Masson, 1988. O.C. Zienkiewicz: The finite element method in engineering sciences, Mac Graw Hill, 1971.
B. Lucquin et O. Pironneau: Introduction au calcul scientifique. Collection Mathématiques appliquées Masson, 1995.

NUMERICAL ANALYSIS II
(25h CM, 20hTD, 15h TP, 10h HP, 4 ECTS)

Educational Purpose: To master the methods for solving the large linear systems resulting from the discretization of PDE's and for calculating the eigenvalues of large dimensional matrices.Basic knowledge of domain decompostion methods.

Prerequisite : Numerical analysis I and Computational tools

Content :

- Methods for eigenvalue computation: power method, reduction of a matrix to a simpler form (Hessenberg, Givens rotation, Householder methods,reduction of a hermitian matrix to tridiagonal form), QR and Davidson-Jacobi methods, Singular value decomposition (symmetric case)
- Krylov accelerated methods:
- ILU preconditioning, geometric and algebraic multigrid preconditioning, deflation methods, domain decomposition methods:
- Primal and dual (Feti) Schur complement methods , Schwarz type domain decompositon methods. Convergence acceleration (ORAS, Aiken-Schwarz), DDM as a preconditioner of Krylov methods (BPS, RAS,...)
- Solving methods for stiff ODE/ADE problems: Shooting methods, Runge Kutta methods, Symmetric integration schemes , symplectic schemes for hamiltonian systems, ADE index , problem-solving techniques.

References:

J.Stoer & R. Bulirsh: Introduction to numerical analysis, Springer text in applied Math. 15

G.H. Golub & C.F. Van Loan: Matrix computation, J. Hopkins University press

E. Hairer & G. Wanner : Solving Differential Equations II, Springer series in Comput. Math 14
Y. Saad : Iterative methods for sparse linear systems, SIAM

UNSTEADY PROBLEMS

(35hCM, 25hTD, 15hTP , 5 ECTS)

Educational purpose : To become conversant with computing techniques, consistency and stability (dispersion/diffusion) analysis and the conservation of schemes derived from a differential operator-based theoretical approach. The discretization schemes used here are essentially restricted to finite difference and volume methods.

Prerequisite : Mathematical methods for engineers I,Basic numerical methods

Content :

a) Numerical schemes for evolution PDE's of first and second order with respect to time. Heat, advection-diffusion, Burgers', wave and reaction-diffusion equations. Finite difference schemes : stability analysis (von Neumann's and Fourier's methods, energy method), dispersion and diffusion analysis. Error and consistency order of finite difference schemes. CFL condition. Upwind schemes.

b) Conservation laws, applications, difficulties, examples : advection, Burgers' and road traffic equations. Systems of conservative laws : wave, Euler, d Navier-Stokes, and Saint-Venant equations. Standard solutions and method of characteristics: linear and non-linear case. Limitations of the method of characteristics which make it necessary to introduce the more general concepts of weak and entropic solutions. Study of a conservation law : differential and integral forms , weak solutions, Rankine-Hugoniot relationships, entropy concept, entropy jump, expansion waves.Riemann problem.

Solving Riemann problem for non linear laws of conservation.

Conservative scheme concept. Conservative methods for non-linear problems : conservative methods, consistency,discrete conservation , Lax-Wendroff theorem,entropy condition. Godunov's scheme. Non linear stability :TVD and monotonic methods.

c)Upwind finite volume methods.

Approximate Riemann solvers : general theory. HLL and HLLC, Rusanov, Roe, Osher, WAF Riemann solvers. Introduction of such methods using a recent more comprehensive and cost-effective PVM approach,.

High order methods and TVD schemes: reconstruction methods, MUSCL approach, generalized Riemann problem, monotonic schemes

and precision, flux and slope limiters. Extension of TVD methods. Source term associated problem. Well balanced schemes.
Support software : Matlab
References :

Semester 10		nb of hours	CM	TD	TP
INTERNSHIP S10					
	30	End of program internship 15 Février 30 Septembre			

* Grégoire Allaire, Analyse numérique et optimisation, Les Editions de l'Ecole Polytechnique, 2006
* Randall J. LeVeque, Numerical methods for conservation laws, Birkhäuser, 1992
* E. Godlewski, P.A. Raviart, Numerical approximation of hyperbolic systems of conservation laws, Applied Mathematical Sciences, 118, Springer, 1996
* **TIME SERIES ET DURATION MODELING**

(30hCM, 20hTD, 10hTPn 10hHP, 4Ects)

TIME SERIES - 20h CM, 14h TD 6h TP
Educational purpose
The course aims at presenting statistical treatment methods related to time series : smoothing,deseasonalization and forecast.By the end of the course students should have developed an awareness of the potential application of time series in industry, in the economy, .. and be able to analyze them using software outputs.
Prerequisite: Random modeling courses of semesters 5 and 6, Regression models.
Content
Descriptive analysis of time series (seasonal decomposition, exponential smoothing) ;
Random modeling of a time series : second order process, stationarity, autocovariance function, autocorrelation function, partial autocorrelation function, spectral density ;
Univaried processes :MA, AR, ARMA, ARIMA, SARIMA ;
Practice of SARIMA models(Box-Jenkin methodology) : identification, estimation, verification, validation, comparison.
Support software : R et SAS
References

¦ Brockwell, P. & Davis R., *Introduction to Time Series and Forecasting*, Springer, 1996.
¦ Bosq D., Lecoutre J-P., *Analyse et prévision des séries chronologiques. Méthodes paramétriques et non paramétriques*,Masson, 1992.
¦ Aragon, Y., *Séries temporelles avec R : Méthodes et cas*, Springer, 2011.

DURATION MODELS - 10h CM, 6hTD, 4hTP

Educational purpose

By the end of the course, students should be familiar with the vocabulary and the tools used in duration modeling as well as with its main applications to reliablity , health care issues,.. They are expected to be familiar with the models and the associated underlying assumptions as well as with the commonly encountered problems. They should also be able to calculate standard estimators and interpret standard tests, using the R and SAS softwares

Prerequisite: Random modeling courses of semesters 5 and 6

Content

Duration models ;
Parametric and non-parametric modeling
Regression models ;
Bayesian approach.

Support software : R or SAS

References

Droesbeke, J.J. & Fichet, B. & Tassi, P., *Analyse Statistique des durées de vie. Modélisation des données censurées*
Klein, J.P.&Moeschberger,M.L., *Survival Analysis : Techniques for Censored and TruncatedData*, Springer, 2003.

Semester 9

MATHEMATICAL MODELING-DISCONTINUOUS GALERKIN

(45HCM ,45 HTP, 5 ECTS)

DISCONTINUOUS GALERKIN METHODS (CM 15, TP 15)

Educational purpose :

By the end of the course, students should be able to discretize PDE systems using the discontinuous Galerkin method and should be conversant with its properties, in particular from a numerical point of view.

Prerequisite : Mathematical methods for engineers , Finite elements I

Content: Definition of the method's specific functional framework . Discontinuous variational formulations, broken Sobolev spaces,non-compliant error analysis. Reminders on orthogonal polynomials(for approximation spaces). DG methods for scalar and non-linear conservation laws : weak formulation, well-posed problem, stability, convergence, error computation, inf sup condition. RKD methods. How to introduce diffusion operators in the DG methods(symmetric interior penalty methods, ...). DG methods will be shown to be mixed methods that are stabilized for diffusion problems. Numerical fluxes approximation:centered and upwind schemes, examples and applications.Use of non conforming meshes. Unsteady problems.Application to Stokes, Navier-Stokes and Saint-Venant equations.

References :

D.A. Di Pietro, A. Ern, Mathematical aspects of discontinuous Galerkin methods, Springer, 2012.

B. Rivière, Discontinuous Galerkin methods for solving elliptic and parabolic equations: Theory and implementation. SIAM, 2008

MODELING (CM 30, TP 30)

Educational purpose :

To acquire a command of all the problems involved from the modeling of a physical problem to its numerical solution and of the underlying mathematical tools.Students are also expected to be able to combine and implement the discretization methods they have been taught during the first two years of their curriculum.

Prerequisite : Courses in Numerical methods

Content:
In this course, the modeling process is tackled all the way from its physical formulation to its numerical solution. The program consists of several independent modules:
- Modeling in biology : reaction-diffusion equations describing animal coat pattern formation. Analysis of solutions.
Solving non linear models using Newton and Gear's methods.
- Modeling of the valuation of financial products (options, futures).The Black and Scholes equations are derived from brownian motion and discretized using finite difference and finite element methods.
- Modeling in fluid mechanics : the main equations (momentum and continuity) are derived from conservation laws. Lagrangian vs. Eulerian description.The turbulence problem.
Application to various problems in geophysics.
- Modeling of dynamical systems, bifurcations, non linear analysis.

Support software : FreeFem++
References : They will be provided during the course in the form of articles in journals, book chapters, etc..

HIGH PERFORMANCE COMPUTING
(35HCM, 35HTP, 4 ECTS)

Educational purpose :
To acquire the reflex of conceptualizing distributed programming, of writing a new module compliant with the standards of a given code, and of validating and verifying a code.
Prerequisite : Numerical analysis I and II, Tools of scientific computing and Software engineering.
Content :
Introduction to message exchange libraries (MPI) ,compilerdirectives programming (OpenMP) and to computation using GPGPU (opencl). Modeling of distributed architectures, code performance assessment for a given architecture.
High performance computing methods suited to the programming models : pipeline, understructuring, domain decomposition (Schur, Feti, Schwarz), decomposition of operators, decomposition in function spaces,distributed DFT, parallel Krylov methods. Code and/or PDE coupling methods.
Code validation and verification procedure .
Software support :
MPI, openMP, Opencl, Fortran90, Petsc, Scalapack.

ADVANCED REGRESSION MODELS
(24h CM, 14h TD, 12h TP, 4 ECTS)
Educational purpose
This course aims at further developing knowledge acquired in the *Regression models* course .Different models will be presented and applied to concrete situations. Students should eventually be able to implement and interpret such models: mixed and generalized linear models .They will also have some basic knowledge of experimental design and of non parametric modeling.
Prerequisite: Random modeling of semesters S5 et S6, Regression models.
Content
Mixed models ;
Experimental designs ;
Generalized linear models , logistic regression in particular (link to the classification) ;
PLS regression, Ridge ;
Non- and semi parametric regression models
Software support : R et SAS
References
Dobson, A.J., *An introduction to generalized linear models*, Chapman & Hall, 1990.
Antoniadis, A. & Berruyer, J. & Carmona, R. *Régression non linéaire et applications.* Economica, 1992.
Verbeke, G. &Molenberghs, G. *LinearMixedModels in practice*, Springer, 1997.
Ruppert,D. & Wand, M. P. & Carroll, R. J., *Semiparametric Regression.* Cambridge Series in statistical
and Probabilistic, 2003.

PROCESS AND RISK STATISTICS
(32h CM, 20h TD, 8h TP)

PROCESS STATISTICS - (20h CM, 12h TD, 8h TP)
Educational purpose
By the end of the course, students should be conversant with the modeling of the main time-dependent random phenomena occuring in industry and biology.... These modelings will be studied both from a probabilistic and a statistical point of view.

Prerequisite Random modeling of semesters S5 et S6.

Content

Introduction to processes ;

Independent increment processes and brownian motion

Poisson process ;

Birth and death process, renewal process ;

Waiting lines

Software support : R

References

Foata,D. & Fuchs, A. , *Processus stochastiques - Processus de Poisson, chaînes deMarkov et martingales, Cours, exercices et problèmes corrigés*, Dunod, 2002.

Bercu, B & Chafai, D., *Modélisation stochastique et simulation*, Dunod, 2007.

RISKs - 12h CM, 8h TD

Educational purpose

How to evaluate the probability of sparse events ?How to determine the height of a dyke for the probability of any overflow to be smaller than some regulatory value?How to set the rate of a reinsurance premium? The course is intended to provide the probabilistic and statistical tools needed to answer the above questions.Attention will also be paid to the numerical implementation of the methods proposed in R.

Prerequisite : Random modeling of semesters S5 et S6.

Content

Introduction to extreme value theory.Examples.

Maximum and threshold overrun modeling .

Evaluation of small probabilities et extreme quantiles .

Sotware support : R

References

Resnick, S. I., *Heavy-tail phenomena. Probabilistic and statistical modeling*, Springer Series in Operations Research and Financial Engineering. Springer, 2007.

De Haan, L. & Ferreira, A., *Extreme value theory. An introduction.* Springer New York, 2006

Beirlant, J. & Goegebeur, Y. & Segers, J. & Teugels, J., *Statistics of Extremes*, Theory and applications. Wiley, 2004.

PROJECT
(15h CM, 15h TD, 50h HP)
Educational purpose
Implementation of the knowledge gained over the course of the program by carrying out of a supervised long term project. Students are offered optional further training in finance or control.

Semester 10